ECON 4643: Development Economics

Lecture 2: Inequality

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Reading

- MB, chap. 10
- DR, Chap. 6-8

Motivation

Inequality

- Questions of this lecture:
- Level of aggregation
- Inequality along which dimension?
- How to measure inequality?
- Why study inequality?

Level of Aggregation

- Between countries
- Within a country
- Among a group of people
- Within a household (a family)

Inequality along which dimension?

- Income
- Should people have access to the same resources? Opportunities?
- How should choices and performances be taken into account when considering access to resources?
- Consider issues of parental influence
- Does inequality serve a purpose or cause incentive to exist?
- Should one care about inequality or something else? e.g., social mobility, lifetime wealth?

Why study inequality?

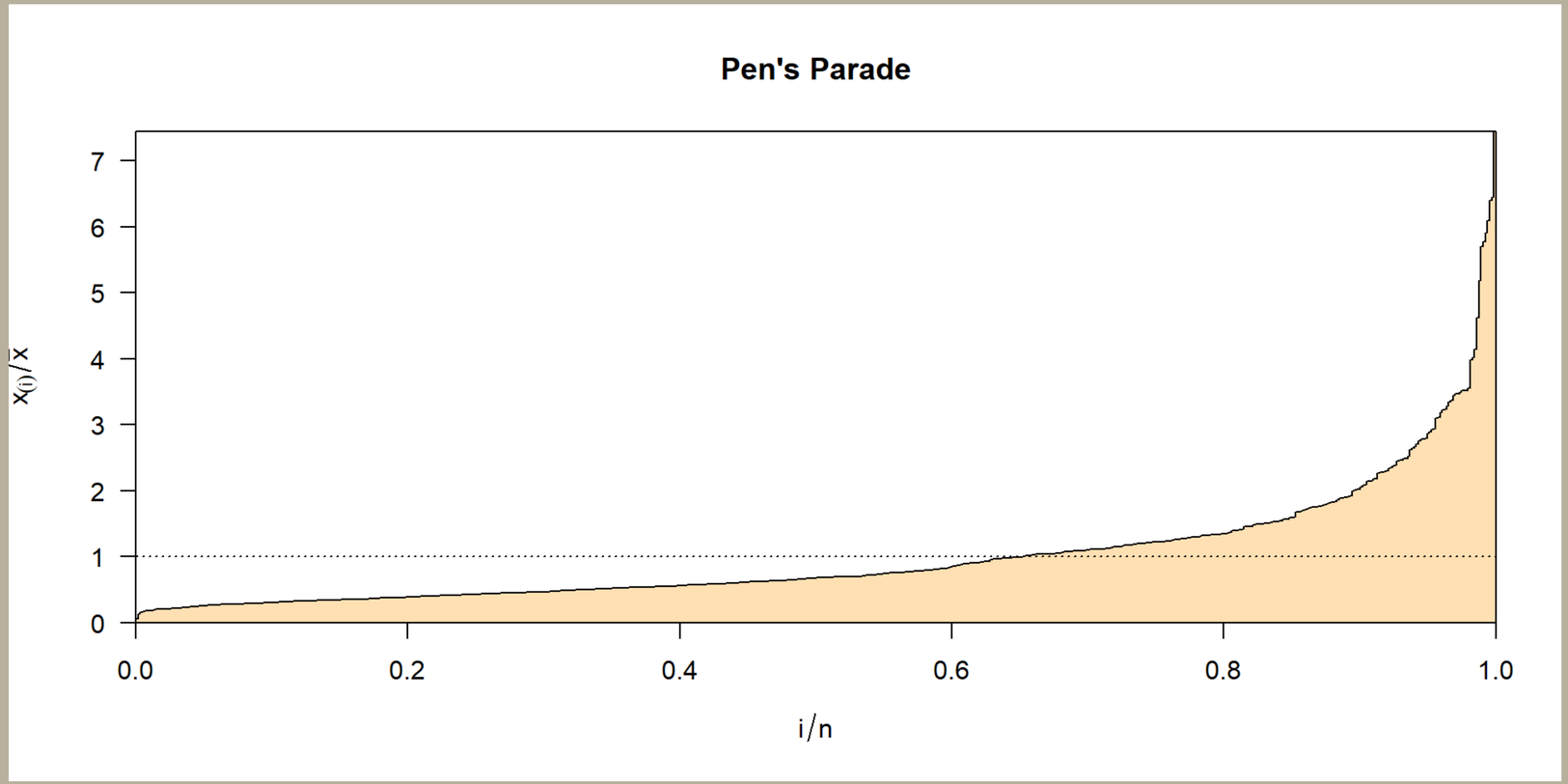
- Political and societal aspects
- Perception of injustice
- Political stability
- Pereception of own economic situation relative to others
- Is equality something that society should want *per se*

Measurement of Inequality

Pen's parade approach

- Graphically depicts how income is distributed across a population, showing the proportion of total income earned by each cumulative percentile of the population.
- Extremes are clear
- But how to comapare in-between cases?

Pen's parade approach: Illustration



Principles of Inequality Measures

Economic Inequality

- Consider two economies $A = (20, 30, 50)$ and $B = (22, 22, 56)$
- Easy to say a society such as $(50, 50)$ is equal and one such as $(0, 100)$ is unequal. What about societies A and B?
- To get an idea of how to compare these two societies we need to agree on some principles that we feel a reasonable measure of inequality should satisfy.
- We are going to discuss four principles that get at *relative* differences in inequality:
- Is A more unequal than B?
- Formalization:
- Let a society have n people
- Income of the n people are $\{y_1, y_2, \ldots, y_n\}$
- Because we are interested in relative inequality we will be comparing $\{y_1, y_2, \ldots, y_n\}$ and $\{y'_1, y'_2, \ldots, y'_n\}$

Anonymity Principle

- Inequality measure should be independent of who owns the income
- Should not mater whether we are looking at a society with $\{100, 200, 300\}$ or $\{300, 200, 100\}$
- Even if one person feels she would prefer a specific society to the other, can we say one of the above society is more unequal?
- This lead to the first principle:
- **Anonymity Principle:** For any distribution of income in a society we can write that distribution as $\{y_1, y_2, \ldots, y_n\}$ where $y_1 \leq y_2 \leq \ldots \leq y_n$.
- For any given society, reordering individuals from the lowest to the highest income does not change inequality in that society.

Population Principle

- This principle says that cloning the population should not alter inequality.
- If we had a society such as $\{100, 200, 300\}$ and cloned each individual so that we had society $\{100, 100, 200, 200, 300, 300\}$ should we consider one of these societies more or less equal than the other?
- This means that all that matters are the proportions of the population that earn different levels of income.
- This leads us to our second principle:
- **Population Principle:** If we compare an income distribution over n people and another population of $2n$ people with the same income pattern repeated twice, there should be no difference in inequality between the two income distributions.

Relative Income Principle

- Consider distributions $\{100, 200, 300\}$ and $\{200, 400, 600\}$. Should we say that one economy is more unequal than the other?
- In terms of development the level of income might matter (such as in the level of per capita income) but not in terms of inequality.
- Inequality represents how people's income are in relative terms. Therefore a measure of inequality should not be subject to the absolute level of incomes.
- Thus the third principle:
- **Relative Income Principle:** It is tantamount to the assertion that income levels, in and of themselves, have no meaning as far as inequality measurement is concerned. Thus the inequality in $\{100, 200, 300\}$ is equal to the inequality in $\{200, 400, 600\}$.

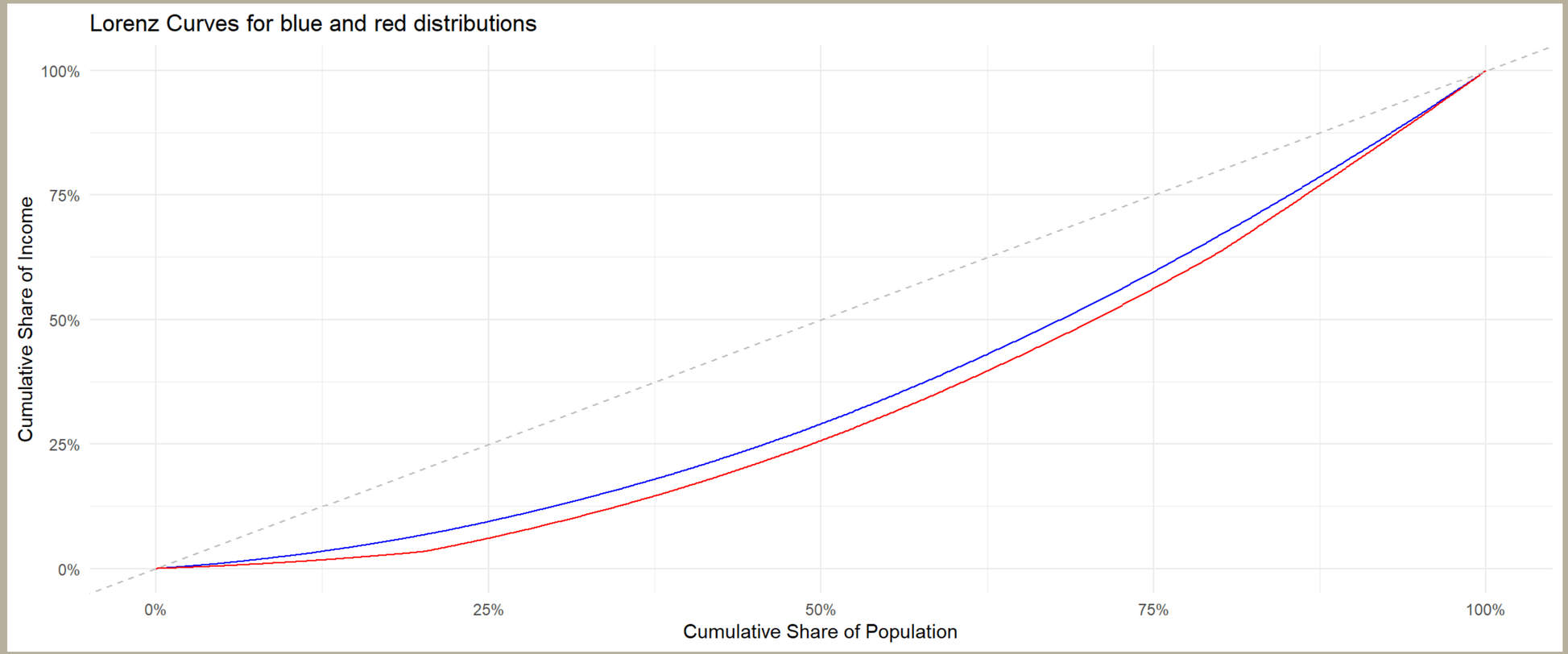
Dalton Principle

- If we take money from someone who is poorer and give it to someone richer in a society what should we say about inequality in that society?
- Example: compare $Y_1 = \{90, 200, 310\}$ to $Y_2 = \{100, 200, 300\}$ Which is more unequal?
- We formalize this idea below:
- **Dalton Principle:** Consider $\{y_1, y_2, \ldots, y_n\}$ such that $y_1 \leq y_2 \leq \ldots \leq y_n$ and consider incomes y_i and y_j such that $y_i \leq y_j$. A transfer of money from y_i to y_j will be called a regressive transfer. If one income distribution Y_1 can be obtained from another income distribution Y_2 through a regressive transfer then Y_1 has a higher level of inequality.
  - In the above example, we can obtain Y_1 by applying using a regressive transfer on Y_2, i.e. transferring $\$10$ from the poorest individual to the richest individual. Therefore, Y_1 is more unequal than Y_2.

Lorenz Curves

- We will look at these 4 principles apply to **Lorenz Curves**
- Consider an income distribution and calculate the following:
  - Percentage Share of total income earned by the bottom $x\%$ of the population. Where $x \in [0, 100\%]$
- If we plot this percentage share on the x-axis with the amount of income that share has on the y-axis we will get the Lorenz Curve of that income distribution.
- **Note:** Perfect equality $\Rightarrow$ 45°-line is the Lorenz Curve.
- **Note:** As our Lorenz curve moves further away from the 45°-line inequality $\Uparrow$

Lorenz curves: Easily Comparable Case

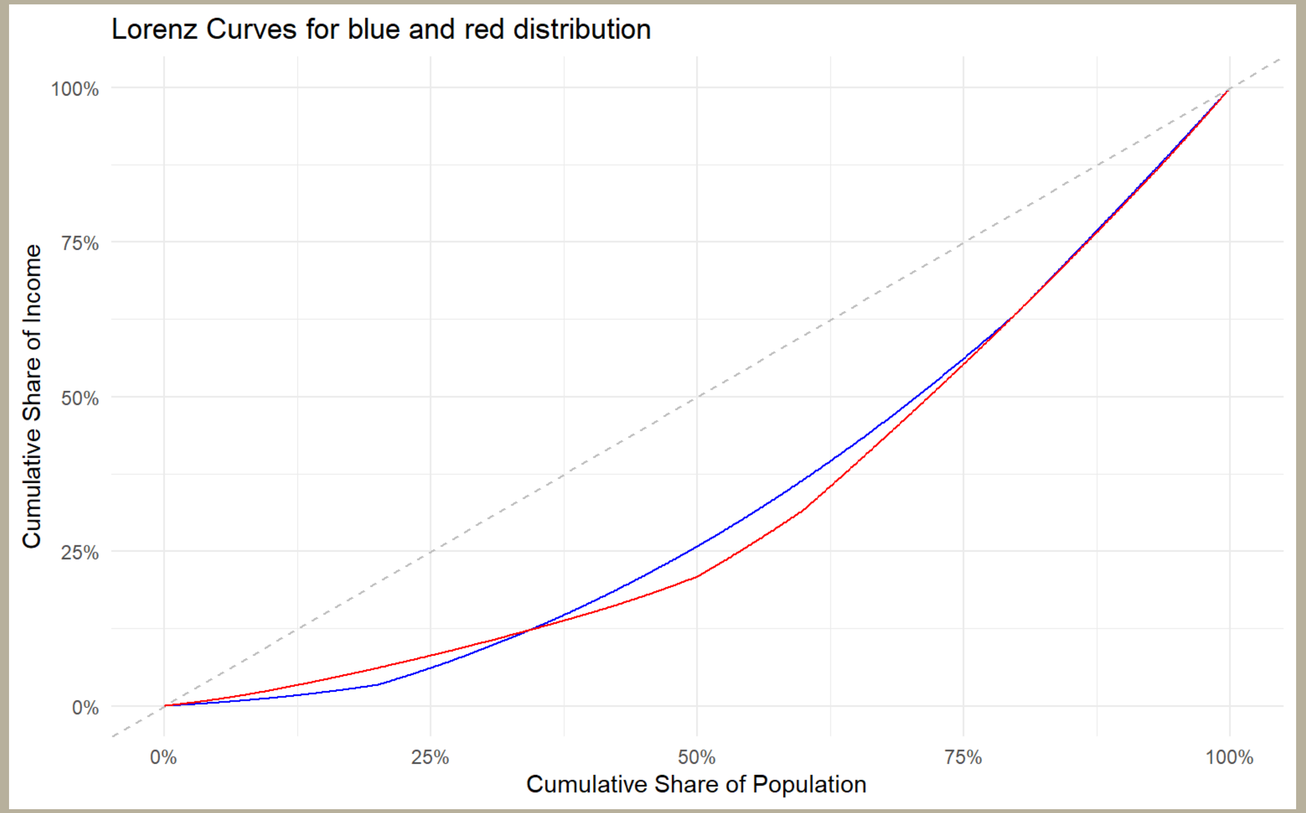


Lorenz Curves

- Lets now compare to income distributions $Y = \{y_1, y_2, \ldots, y_n\}$ and $Y' = \{y'_1, y'_2, \ldots, y'_n\}$
- Say that the Lorenz curve associated with income distribution Y is L_Y and the Lorenz curve associated with income distribution Y' is $L_{Y'}$.
- We say an income distribution Y' is more unequal compared to Y if $L_{Y'} \leq L_Y$ for all $x\% \in [0, 100\%]$.
- We can see this easier if we graph both curves and will do so shortly.
- Note that if $L_{Y'}$ is below $(\leq) L_Y$ for all values of $x\%$ then the poorer people in L_Y, have less of a share of the income when compared to the same $x\%$ of the population in L_Y.
- We say an inequality measure is Lorenz-Consistent for Y and Y' it is true that $L_{Y'} \leq L_Y$ for all $x\% \in [0, 100\%]$.
- **Theorem:** An inequality measure is Lorenz-Consistent if and only if it is simultaneously consistent with the anonymity, population, relative income and Dalton principles.

- Lets look at how the Dalton Principle is satisfied with the Lorenz criteria.
- If money is taken from someone who has income $y_j \leq y_i$ then the cumulative income held by the people $1, 2, \ldots, i$ is less then before and the cumulative income held by the people $j, j+1, \ldots, n-1, n$ is more.
- Therefore the Lorenz curve after this regressive transfer should move further away from the 45°-line.
- In the previous figure:
  - The distribution in *blue* is more equal than the one in *red*
  - The distribution in *red* is obtained from the one in blue by transferring income from the bottom 20% to the top 80% of the income distribution, i.e. by applying a regressive transfer
- Unfortunately, though, the Lorenz curve can lead to lots of cases where we cannot compare inequality within two countries.
- If two Lorenz curves cross $\Rightarrow$ No clear comparison can be made regarding inequality.
- If there exists some $x\%$ such that $L_1(x) > L_2(x)$ and there exists some $y\%$ such that $L_1(y) < L_2(y)$ then we say that the Lorenz curves for income distributions 1 and 2 cross.
- In this case we cannot go through a series of transfers from one economy to the next, using our set of principles, to arrive at a conclusion of which income distribution is more unequal.
- Consider the following graph.

Lorenz Curves: Inconclusive Case



The Gini Coefficient

- When Lorenz curves bring ambiguous results many people turn to the Gini coefficient.
- The Gini coefficient is a complete ranking - it spits out a number for every conceivable distribution.
- Besides the Gini coefficient there are other complete rankings we can look at; such as the range, Kuznets ratios, mean absolute deviation, and coefficient of variation.
- We need more formality before discussing these other measures, though. Say:
- There are m distinct incomes
  - n_j people are earning income in class j
  - n is the total number of people and is $n = \sum_{j=1}^{m} n_j$
  - μ is the mean of a distribution and is defined $\mu = \frac{1}{n} \sum_{j=1}^{m} n_j y_j$

The Gini Coefficient

-We can now define the Gini-coefficient:

$$G = \frac{1}{2n^2\mu} \sum_{i=1}^{m} \sum_{j=1}^{m} n_i n_j \left| y_i - y_j \right|$$

- The Gini is actually defined on the sum of all pair-wise comparisons of income in the population.
- The Gini-coefficient satisfies all four of the properties we checked earlier.
- There is also a graphical interpretation that may help you to remember the Gini-coefficient:
- It is the ratio of the area between the Lorenz curve and the 45°-line and the area under the 45°-line as a whole.
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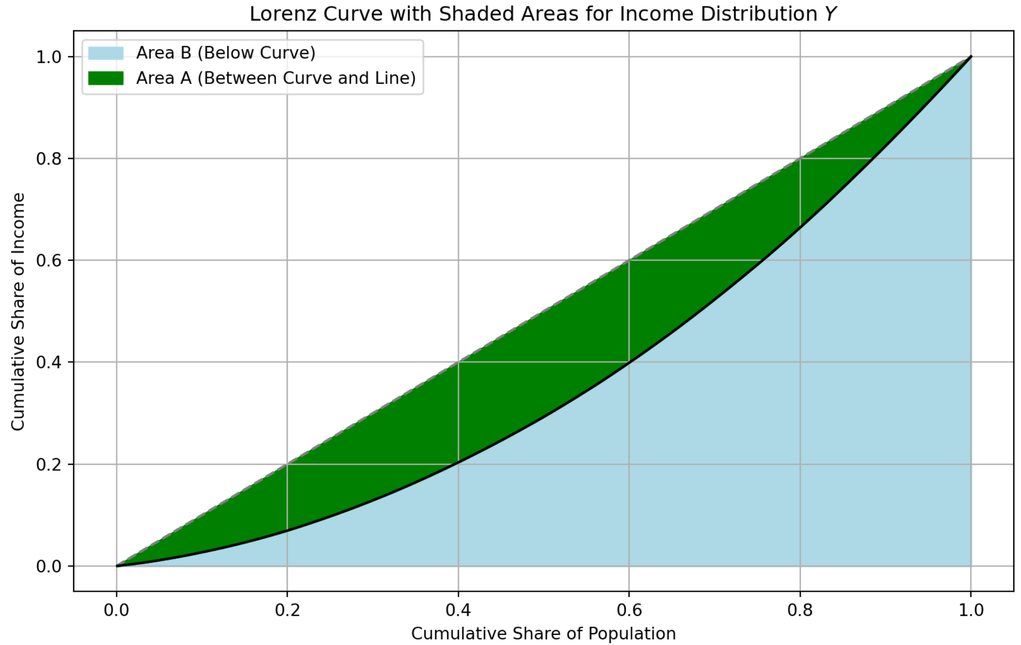
The Gini Coefficient

- Gini-Coefficient can be thought of as $G = \frac{A}{A+B}$ where A and B are as illustrated below.

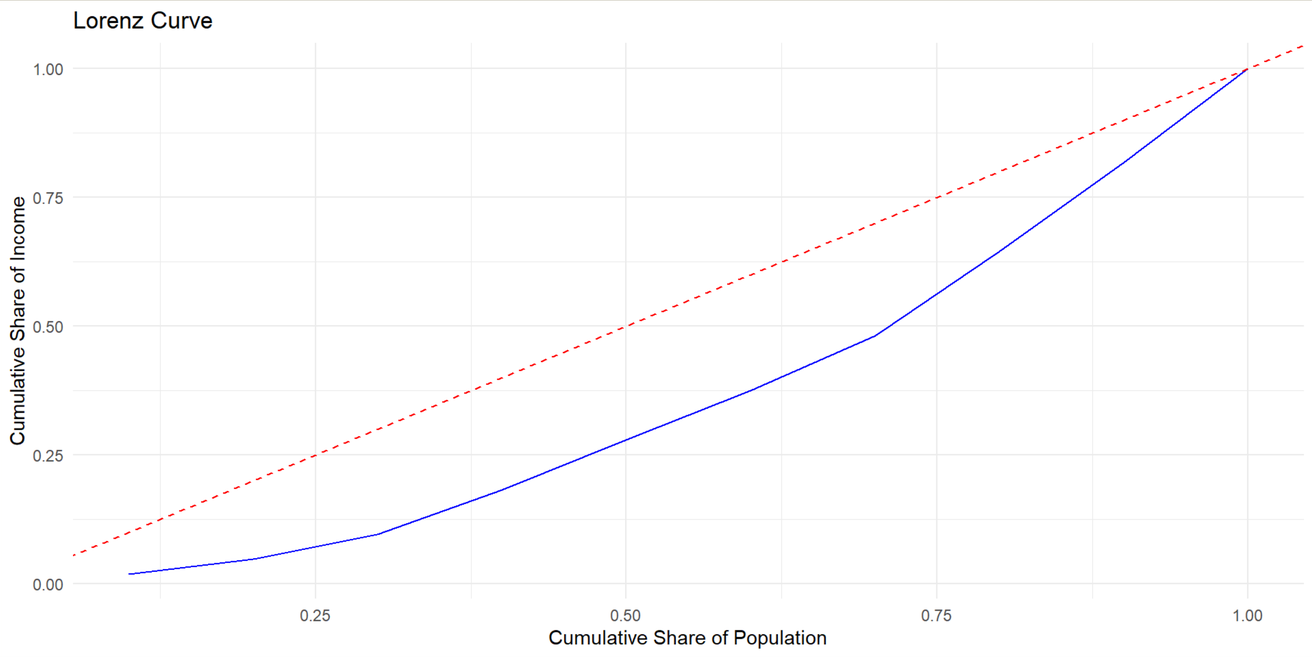


Illustration

- Graph the Lorenz curve, using 10 observations

```r
# Load ggplot2 library
library(ggplot2)

# Income data
income <- c(9, 18, 10, 10, 19, 3, 2, 5, 11, 17)

# Sort the income data in ascending order
income <- sort(income)

# Calculate the cumulative share of income
cum_income <- cumsum(income) / sum(income)

# Calculate the cumulative share of the population
pop <- 1:length(income) / length(income)

# Create a data frame for plotting
df <- data.frame(pop, cum_income)
```

```r
# Plot the Lorenz Curve using ggplot2
ggplot(df, aes(x = pop, y = cum_income)) +
  geom_line(color = "blue") +
  geom_abline(slope = 1, intercept = 0, color = "red", linetype = "dashed") +
  xlab("Cumulative Share of Population") +
  ylab("Cumulative Share of Income") +
  ggtitle("Lorenz Curve") +
  theme_minimal()
```



Illustration

- Gini Coefficient, using 10 observations.
  - We apply

$$G = \frac{1}{2n^2\mu} \sum_{i=1}^{m} \sum_{j=1}^{m} n_i n_j \left| y_i - y_j \right|$$

```r
# Income data
income <- c(9, 18, 10, 10, 19, 3, 2, 5, 11, 17)

# Calculate mean income (mu)
mu <- mean(income)

# Number of entities (n)
n <- length(income)

# Initialize sum of absolute differences
sum_abs_diff <- 0

# Compute the double summation of absolute income differences
for (i in 1:n) {
  for (j in 1:n) {
    sum_abs_diff <- sum_abs_diff + abs(income[i] - income[j])
  }
}
```

```
[1] 0.3115385
```

Other complete measures of inequality

- **The Range:**

$$R = \frac{1}{\mu}(y_m - y_1)$$

- **Kuznets ratios:** ratio of the shares of income of the richest $x\%$ to the poorest $y\%$ where x and y stand for numbers such as 10,20 , or 40. Example given, the $\frac{20}{40}$-ratio.
- **Mean Absolute Deviation:**

$$M = \frac{1}{\mu n}\sum_{i=j}^{m} n_j \left|y_i - \mu\right|$$

- **Coefficient of variation:**

$$C = \frac{1}{\mu}\sqrt{\sum_{j=1}^{m}\frac{n_j}{n}(y_j - \mu)^2}$$

Why Study Inequality

- If we do not care about inequality itself we may care about how it *affects* income or growth.
- Savings rates are effected by income levels.
- Inequality could provide incentives for people to work harder to reach a different social status.
- Access to credit and finance is constrained.
- We must focus on the causal story, i.e. does inequality cause…?
- We will look at the Kuznet's curve and then four potential models of why income inequality and growth may be related.

Inequality and Development: The Kuznets Hypothesis

- Kuznets hypothesized that there was an inverted-U shaped relationship between inequality and development.
- Kuznets suggested that: economic progress, measured by per capita income, is initially accompanied by rising inequality, but these disparities ultimately go away as the benefits of development permeate more widely. -Low-level of income $\Rightarrow$ Low inequality.
  - Moderate-levels of income $\Rightarrow$ High inequality.
  - High-levels of income $\Rightarrow$ Low inequality.
- Two ways to test this relationship:
- Look at countries over a long period of time: Time Series.
- Look at a group of countries at one point in time: Cross-Section.
- We will use a cross-sectional approach.

Inequality and Development: The Kuznets Hypothesis

- How to test Kuznets' hypothesis?
- Let s_i be the share of total income going to group i.
- Let y be the level of per capita income.
- As $y \Uparrow$ we expect that the share of income held by the richest 20%, s_{20} will at first $\Uparrow$ and then $\Downarrow$.
- As $y \Uparrow$ we expect that the share of income held by the poorest 20% s_p will at first $\Downarrow$ and then $\Uparrow$.
- To test this relationship we can use the following regression:

$$s_i = A + by + cy^2 + D + \varepsilon$$

- What signs should each of the coefficients take?
- Ahluwalia analyzed a sample of sixty countries: 40 developing; 14 developed; 6 socialist.
- Ahluwalia used the above equation to test the Kuznets inverted-U hypothesis… What do you think he found?

Inequality and Development: The Kuznets Hypothesis

Ahluwalia' results

Income Share	y	y^2	**Socialist Dummy**	R^2
Top 20%	89.85 (4.48)	17.56 (4.88)	-20.15 (6.83)	0.58
Middle 40%	-45.59 (3.43)	9.25 (3.88)	8.21 (4.20)	0.47
Lowest 20%	-16.97 (3.71)	3.06 (3.74)	5.54 (8.28)	0.54

- Ahluwalia found that the Kuznets hypothesis holds.
- *What do you think about these results?*
- *What else could be done to test the inverted-U hypothesis?*
- *Do you believe Ahluwalia's results?*
- There are many data examples in the book BUT the general message is that the Kuznets' inverted-U hypothesis probably does not hold except in a small number of cases.